| Indian Institute of Technology Palakkad भारतीय प्रौद्योगिकी संस्थान पालक्काड <br> Nurturing Minds For a Better World |  |  |  |
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| MATHEMATICS DEPARTMENTAL SYMPOSIUM - 2024 |  |  |  |
| 19th Jan'24 |  | 20th Jan'24 |  |
| 9:00-9:05 | Welcome - M Ashok Kumar |  |  |
| 9:10-10:00 | Praveen Chandrashekar, TIFR-CAM <br> Shear shallow water model and its numerical approximation | 9:00-9:50 | H. Ananthnarayan, IIT Bombay <br> Insolvability of the Quintic |
| 10:10-10:40 | Jaikrishnan J, IIT Palakkad <br> Conformal mapping through the lens of invariant metrics | 10:00-10:30 | R Venkatesh, IIT Palakkad/IISc Bangalore <br> Some applications of graph polynomials in number theory |
| Tea Break |  | Tea Break |  |
| 11:00-11:50 | Kaneenika Sinha, IISER Pune <br> Two perspectives in number theory: explicit and probabilistic | 11:00-11:50 | Divyang Bhimani, IISER Pune <br> Integrating harmonic analysis, dispersive PDEs and Probability |
| 12:00-12:50 | Gopikrishnan C R, IIT Palakkad <br> Unified a priori analysis of four second-order FEM for fourth-order quadratic semilinear problems | 12:00-12:50 | V Muruganandam, IIT Palakkad <br> The Legacy of Harish-Chandra - A bird's view |
| LUNCH |  | LUNCH |  |
| 2:00-2:50 | Debraj Das, IIT Bombay <br> Central Limit Theorem in High Dimension | 2:00-3:30 | Discussion Session |
| ${ }^{1} 3: 00-3: 25$ | Jiju Mammen, IIT Palakkad <br> Exploring Holomorphic Retracts |  | Vote of thanks: Sarath Sasi |

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## ABSTRACTS:

## H. Ananthanarayanan

Title: Insolvability of the Quintic


#### Abstract

How do you show something cannot be done? The insolvability of the quintic is one such problem. In this talk, we will understand what one means by "solvability", and how one can answer the question by transferring the problem to a different domain. We will see this process via examples, and end by giving an overview of how the problem was finally solved.


## Debraj Das

Title: Central Limit Theorem in High Dimensions


#### Abstract

Central Limit Theorem (CLT) is one of the oldest as well as remarkable results of classical probability theory. In most simplest words, CLT is a statement about the convergence of properly centered and scaled sample mean of a sequence of random vectors to the Gaussian random vector in distribution. Although most of the theoretical developments centred around to establish CLT whenever the underlying dimension of the random vector is fixed, the recent interest lies in establishing CLT when the dimension also grows with the sample size. I will shed light on recent developments as well as describe some results related to critical growth rate of dimension. If time permits, I will briefly describe the benefits of self-normalization /studentization in reducing the requirement of existence of exponential moments to some polynomial moments.


## Divyang Bhimani

Title: Integrating harmonic analysis, dispersive PDEs and Probability

Abstract: We shall see how modern harmonic analysis and probability techniques are helpful to answer and understand some of the fundamental questions for the nonlinear Schrodinger equation (a particular example of dispersive PDEs).

## Gopikrishnan C R

Title: Unified a priori analysis of four second-order FEM for fourth-order quadratic semilinear problems


#### Abstract

A unified framework for lowest-order finite element schemes that discretises fourth-order problems with semilinear nonlinearity is discussed. Examples of such problems are stream vorticity formulation of the Navier-Stokes equation and von-Karman equations. We provide stability and a priori energy estimates in weaker Sobolev norms under minimal hypotheses. Popular finite element schemes covered by this framework are Morley, discontinuous Galerkin method, and CO-interior penalty method, and weakly over penalized symmetric interior penalty method.


Title: Conformal mapping through the lens of invariant metrics


#### Abstract

A surprising non-classical theorem about conformal mappings proven only in 1978 states that a holomorphic self-map of a bounded planar domain that has 3 fixed points must be the identity. In this talk, I will introduce the famous Kobayashi and Caratheodary invariant distances and give a new and simple proof of the aforementioned theorem using the fact that the balls under these distances are finitely-connected. This is joint work with my PhD student Bharathi T.


## Jiju Mamman

Title: Exploring Holomorphic Retracts


#### Abstract

A fundamental question in complex analysis in both one and higher dimensions is about determining those subsets $S$ of a given domain $D$ in $C^{N}$, which arise as fixed-point sets of some holomorphic self-map of $D$. This being a broad and rather general challenging problem, we consider the following sub-problem in this talk: determine those subsets of domain of holomorphy $D$ which can be realized as holomorphic retracts; to begin with, balanced pseudoconvex domains $B \subset C^{N}$. Specifically, we show that every (holomorphic) retract passing through its center (i.e., the origin), is the graph of a holomorphic map over a linear subspace of $B$. As for retracts not passing through the origin, we obtain the following result: if $B$ is a strictly convex ball with respect to some norm on $C^{N}$, and $\varrho$ any holomorphic retraction map on $B$ which is submersive at its center, then $Z=\rho(B)$ is the graph of a holomorphic map over a linear subspace of $B$. To deal with a case when $\partial B$ may fail to have sufficiently many extreme points, we consider products of strictly convex balls, with respect to various norms and obtain a complete description of retracts passing through the origin. The conditions of this result can indeed be verified to be applied to solve a special case of the union problem with a degeneracy, namely: to characterize those Kobayashi corank one complex manifolds $M$ which can be expressed as an increasing union of submanifolds which are biholomorphic to a prescribed homogeneous bounded balanced domain in $\mathrm{C}^{N}$. Along the way, we discuss examples of non-convex but pseudoconvex bounded balanced domains in which most one- dimensional linear subspaces fail to be holomorphic retracts, in good contrast to the convex case. Indeed, for the simplest possible case here namely the ' $\ell_{q}$-ball' for 0 < $q<1$, we obtain a complete list of all retracts through the origin. To go beyond balanced domains, we then first obtain a complete characterization of retracts of the Hartogs triangle. Thereafter, we drop the assumption of boundedness as well, to attain similar characterization results for domains which are neither bounded nor topologically trivial; indeed, for domains with a wide variety of fundamental groups. We conclude by reporting some results on the retracts of $\mathrm{C}^{2}$.


## Kaneenika Sinha

## Title: Two perspectives in number theory: explicit and probabilistic

Abstract: "Explicit" number theory is the name given to the study of what are called zero-free regions of the Riemann zeta function and other L-functions. An explicit determination of such regions often reveals deep arithmetic properties of the underlying object attached to the concerned L-function. More generally, it could refer to the use of "explicit", often technical methods to understand an arithmetic object. On the other hand, "probabilistic" number theory attempts to investigate arithmetic properties of an object by treating the object as one in a family of many, and exploring these families of varying sizes through the
viewpoint of probability. In this talk, we will explore both perspectives and compare the wealth of information each perspective presents to us.

## Praveen Chandrashekar

Title: Shear shallow water model and its numerical approximation


#### Abstract

Shallow flows are common in nature, as in rivers and large scale flow in sea/oceans. The classical shallow water model is derived under the assumption of zero shear and leads to a non-linear system of hyperbolic PDE. Many flows have small but non-negligible shear in which case the solutions of the classical model give poor approximations. Recently, shallow models have been derived that include weak shear effects and are able to correctly model many observed phenomena better than classical models. These models turn out to be more non-conservative and their numerical approximation poses some challenges. We will discuss some work on the construction of numerical methods for the shear shallow water model and their performance in some representative problems.


## Varadharajan Muruganandam

Title: The Legacy of Harish-Chandra - A bird's view


#### Abstract

This year marks the birth centenary of Harish-Chandra. We, in this lecture, make a modest attempt to expose the gist of his profound contributions to the Harmonic analysis of Lie groups which created a legacy for researchers that continues today.

It is an expository talk keeping the students in mind. No prerequisite is expected from the students. Enough care is taken in choosing the contents so that the students can understand the natural flow of the talk and appreciate it.


## R Venkatesh

Title: Some applications of graph polynomials in number theory


#### Abstract

There are various applications of graph polynomials in algebra and number theory. We mainly focus on one such instance. Let G be a simple finite graph. Radchenko and Villegas [Bull. London Math. Soc. 2021] gave an interesting characterization of chordal graphs, namely $G$ is chordal graph if and only if the inverse of the multi-variate independence polynomial of G is Horn hypergeometric. I will explain how to get a simpler proof of this fact using the connection between the inverse of the multi-variate independence polynomial of G and multi-colored chromatic polynomials of G . This is a joint work with Dipnit Biswas and Irfan Habib.


[^0]:    ${ }^{1}$ Registration link: https://forms.gle/ozYhoiXKx47P8faz8

