

ERROR IN “WIENER’S THEOREM, INFINITE MATRICES AND BANACH ALGEBRAS”

S. H. KULKARNI

There is an error in Remark 3.7 of the article [2] “Wiener’s Theorem, Infinite Matrices and Banach Algebras” that appeared in the last issue (Vol. 19, No. 2) of the Mathematics Newsletter of the Ramanujan Mathematical Society. The Remark deals with the algebras A and B , where $A = C(\Gamma)$, is the set of all continuous complex valued functions on the unit circle Γ with the supremum norm and B is the set of functions in A with absolutely convergent Fourier series. It is then said that B is a closed subalgebra of A . This is wrong, B is not a closed subalgebra of A . (In fact, if this were the case, then B would coincide with A by the famous Stone-Weierstrass Theorem.) Thus the inverse-closedness of B does not follow from Theorem 3.6 and this is the error.

However, the inverse-closedness of B follows from Wiener’s Theorem. As mentioned in the Introduction of [2], Gelfand gave an elegant proof of Wiener’s theorem using the techniques from Banach Algebras. Gelfand’s proof can be found in many books, for example, [1], [6]. In the expository article [3], this proof is presented along with some basic minimum facts from the theory of Banach algebras required to understand this proof. Several proofs of Wiener’s theorem are available in the literature. Some of these can be found in [5]. Among these there is a very simple, short and elementary proof due to Newman [4].

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DEPARTMENT OF MATHEMATICS, INDIAN INSTITUTE OF TECHNOLOGY - MADRAS,
CHENNAI 600036
E-mail address: `shk@iitm.ac.in`